

Ratti & McWaters/College Algebra and Trigonometry, Pearson, 2009

A

- AAS triangles, 629
 - area of, 653–654
 - solving, 631–634
- ABC length formula, 486
- Absolute value, 658
 - of complex number, 696, 697
 - defined, 8
 - equations involving, 157–159
 - inequalities involving, 159–161
 - properties of, 9
- Absolute value bars, 9
- Absolute value function, 244, 245–246
- AC (alternating current), 591–592
- Acceleration, 386
- Accident investigation, 376, 378
- AC circuits, 108, 114, 695, 699–700
- Action-at-a-distance force, 658
- Acute angle, 71, 480
 - trigonometric functions of, 492–495
- Adams, Evelyn Marie, 945
- Addition
 - of complex numbers, 110–111
 - of fractions, 13–14
 - matrix, 801–802
 - of polynomials, 31
 - of radical expressions, 64–65
 - of rational expressions, 51–54
 - with different denominators, 53
 - with same denominator, 51
 - when denominators have no common factor, 52
 - of vectors, 659–660, 662
- Addition method. *See* Elimination method
- Additive identity, 10
- Additive Identity Property, 803
- Additive inverse, 11
- Additive Inverse Property, 803
- Additive Rule, 948–949
- Agreement on domain, 215
- Air navigation, vectors in, 666
- Algebra
 - Fundamental Theorem of, 352
 - matrix (*See* Matrix algebra)
- Algebraic, 424
- Algebraic expressions, 14–16
 - value of, 14
- Algebraic vectors, 660–662
 - arithmetic operations and properties of, 662
- Al-Khwarizmi, Muhammad ibn Musa, 712
- Alternating current (AC) circuits, 591–592, 695, 699–700
- Altitude, of triangle, 630
- Ambiguous case, 637–639
- American Journal of Mathematics*, 801
- Amperes (amps), 587
- Amplitude, 524–527
- Angle(s), 479–480
 - acute, 71, 480
 - trigonometric functions of, 492–495
 - central, 482–483
 - complementary/complement, 71, 485, 497
 - coterminal, 480, 509
 - of depression, 499
 - determining quadrant in which it lies, 510
 - direction, 664–665
 - of elevation, 499–501
 - frequently used, 485
 - multiple, 614
 - negative, 480
 - obtuse, 71
 - positive, 480
 - quadrantal, 508–509
 - reference, 510–511
 - using, 511–514
 - right, 480
 - rotation, 555
 - straight, 480
 - supplementary/supplement, 71, 485
 - trigonometric equations involving multiple, 614–617
 - trigonometric function values of common, 496
 - between two vectors, 671–673
 - uses of, 478
- Angular speed, 487–488
- Annual rate, 417
- Annual yield, 427
- Annuities, 914–916
- Aphelion, 868
- Apollonius of Perga, 846
- Application(s). *See also* Trigonometric function
 - applications
 - composition of functions, 272–274
 - ellipses, 865–866
 - exponential equations, 459–461
 - Factor Theorem, 336
 - finance, 39, 43–44, 98–99, 437–438, 738, 741–742
 - functions, 220–221
 - geometric sequences, 914–916
 - geometry, 97
 - hyperbola, 880
 - interest, 98–99, 417–421
 - inverse functions, 286
 - linear equations, 96–107
 - linear programming, 760–761
 - matrix inverses, 820–821
 - mixtures, 103
 - one-to-one functions, 286

- quadratic equation, 126–127
- quadratic functions, 305
- right triangle trigonometry, 499–501
- solving applied problems, 96, 97
- systems of linear equations in two variables, 719
- systems of nonlinear equations, 741–742
- uniform motion, 99–101
- vectors, 665–667
- work rate, 101–103
- Approximation, rational, 4
- Arc, of circle, 485–486
- Arccosine, 549, 553
- Arcsine, 547–548, 553
- Area
 - of sector, 488
 - of a triangle, 651
 - AAS, 653–654
 - ASA, 653–654
 - SAS, 651–653
 - SSS, 654–655
- Area formulas, 74
- Area number, in Social Security number, 934
- Arithmetic expressions, 9–10
- Arithmetic mean, 909
- Arithmeticonum Libri Duo* (Maurolico), 923
- Arithmetic sequences (arithmetic progressions)
 - common difference of, 903, 905
 - defined, 903
 - n th term of, 904–905
 - recursive definition of, 904
 - sum of, 905–907
- Ars Magna* (Cardano), 351, 358
- Art, determining age of work of, 444, 452–453
- Arterial blood flow, 251, 261
- ASA (angle-side-angle) Theorem, 72
- ASA triangles, 629
 - area of, 653–654
 - solving, 631–634
- Associative Property
 - of addition, 245, 803
 - of multiplication, 245, 808
 - of scalar multiplication, 803, 808
- Asymptote(s)
 - of hyperbola, 874–875
 - of rational functions
 - horizontal, 364–366
 - oblique, 369–370
 - vertical, 361–364
- Augmented matrix, 785–786
 - compared to linear system, 788
 - reduced row-echelon form, 789, 792–795
 - row-echelon form, 789–792
- Automobile industry, computer graphics and, 800
- Average cost, 222
- Average rate of change, 234–235
- Average speed, 487
- Axis
 - of cone, 846
 - conjugate, of hyperbola, 871, 872, 877
 - imaginary, 695
 - major, of ellipse, 859, 860, 861, 863
 - minor, of ellipse, 859, 860, 861, 863
 - polar, 680
 - real, 695
 - of symmetry, 182
 - of parabola, 301, 849, 850, 853
 - transverse, of hyperbola, 871, 872, 877
 - x - (See x -axis)
 - y - (See y -axis)
- B
- Bacterial growth, 413
- Ball attached to spring, simple harmonic motion of, 534–535
- Bar graph, 173
- Base, of integer exponent, 19
- Baseball, applying distance formula to, 174–175
- Bearings, using in navigation, 633–634
- Beat frequencies, 602
- Bermuda Triangle, 146, 149, 651
- Bernoulli's equation, 59
- Bernoulli, Jakob, 680
- Bernoulli, Johann, 421
- Binary operations, 2–3
- Binomial, 30
- Binomial coefficients, 927, 928–932
- Binomial expansions, 927–928
 - finding particular coefficient in, 931–932
- Binomial power, expanding
 - using Binomial Theorem, 931
 - using Pascal's triangle, 929
- Binomial sum or difference
 - cubing, 36
 - squaring, 34–35, 36
- Binomial Theorem, 927, 930–932
 - expanding binomial power using, 931
 - finding particular term in binomial expansion, 931–932
- Bioavailability, 210, 220–221
- Bionics, 680, 684–685
- Birthday problem, 955
- Blood pressure measurement, 251
- Blue moon, 613
- Book of Calculators (Liber abaci)* (Fibonacci), 895
- Bouguer-Lambert Law, 474
- Boundary, 746–747
- Bounds
 - on real zeros, 344–345
 - rules for, 344
- Boyle's Law, 393, 880
- Braces, 9

Brackets, 9
 Branches, of hyperbola, 870
 Breaking even, 222–223
 Breakpoints, 246
 Briggs, Henry, 435
 British thermal units (BTU), 336
 Building partitioning, 126–127
 Bungee-jumping, 83, 91

C
 Calculators. *See also* Graphing calculators
 continuous compound interest on, 422
 converting between decimal degree notation
 and DMS form, 482
 evaluating expressions on, 21
 factorials on, 896
 inverse trigonometric functions on, 551–553
 logarithms on, 436, 449
 trigonometric function values on, 496
 Candy consumption, 19, 26
 Carbon emissions, 465–466
 Cardano, Gerolamo, 351, 358
 Cardioid, 688, 690, 692
 Carrying capacity, 456
 Carter, Howard, 444
 Cartesian plane. *See* Coordinate plane
 Catenary curve, 309
 CAT scans, 725, 732–733
 Cauchy, Augustin-Louis, 827
 Cayley, Arthur, 814
 Ceiling integer function, 250
 Cell phone towers, finding triangular area
 determined by, 652–653
 Celsius
 conversion of cricket chirps to degrees, 2, 15
 conversion to Fahrenheit, 90–91, 153
 Center
 of circle, 185
 of ellipse, 859, 860, 861, 863
 of hyperbola, 871, 872, 877
 Central angle, 482–483
 Certain event, 947
 Change in y per unit change in x , 193
 Change of base, 449–450
 Charles's Law, 399
 Chemical toxins in lake, 439–440
 Chia Hsien, 928
 Cholesterol-reducing drugs, 221
 Chord table, 564
 Circle(s), 692, 846, 847
 area of a sector, 488
 defined, 185
 as ellipse, 865
 equation of, 185–188
 formulas, 74
 inscribed, 657
 length of arc of, 485–486
 unit, 514–517
 Circular functions, 514, 516–517
 Circumference formulas, 74
 Closed interval, 7
 Closure Property, 10, 12
 Coding matrix, 822
 Coefficient, 29, 30
 binomial, 927, 928
 leading, 30, 311
 of monomial, 36
 Coefficient matrix, 785–786
 Cofactors, 828–829
 expanding by, 830
 Coffee consumption, 19, 26
 Cofunction identities, 497, 577–578, 594
 Coin toss, 945, 947
 Collinear, 836
 Column matrix, 785
 Column method
 of adding polynomials, 31
 of multiplying polynomials, 33
 of subtracting polynomials, 32
 Combinations, 938–940
 choosing, 941
 Combining functions, 267–269
 Comets, 880
 Halley's, 865–866, 868
Commentarii Academia Petropolitanae (Euler),
 212
 Common difference, 903, 905
 Common factor, 39, 40
 Common logarithms, 435–436
 Common ratio, 910–911
 Commutative Property, 12
 of addition, 245, 803
 Complementary angles (complements), 71, 485,
 497
 Complement of an event, 950–951
 Completing the square, 121–123
 Complex Conjugate Product Theorem, 113
 Complex conjugates, 112–114
 Complex fractions, 54–55
 Complex numbers, 108–110
 absolute value of, 696
 addition of, 110–111
 conjugate of, 112
 complex, 112–114
 division of, 113–114
 complex conjugates and, 112–114
 equality of, 110
 geometric representation of, 695–696
 multiplication of, 111–112
 by conjugate, 113
 polar form of, 696–698
 power of, 700–701

- product and quotient in, 698–700
 - roots of, 701–703
 - subtraction of, 110–111
- Complex plane, 695
- Complex polynomial, 351–352
- Complex rational expression, 54–55
- Complex zeros of polynomial functions, 351–358.
 - See also Real zeros of polynomial functions
 - Conjugate Pairs Theorem, 353–356
 - defined, 352
- Components of vector, 661, 676–677
- Composition of functions, 269–271
 - applications, 272–274
 - domain of, 271–272
 - trigonometric and inverse trigonometric, 553–555
- Compounded annually, 419
- Compound inequality, 150–152
- Compound interest, 418–421
 - continuous compounding, 422–423
 - formula, 420–423, 914
- Compressing a function, 257–259, 260
 - exponential functions, 411
- Computer-aided design (CAD), 800
- Computer-aided manufacturing (CAM), 800
- Computer graphics, 800, 808–809
- Condensed form, of logarithmic expression, 446, 447
- Conditional equation, 84, 87
- Conditional inequality, 147
- Conditional linear equation in one variable, 85
- Conditional rational equation, 88
- Cone(s)
 - right circular, 846
 - sonic, 544
- Congruent triangles, 72, 629
- Congruent-triangle theorems, 72
- Conic sections (conics), 845–890
 - degenerate, 847
 - ellipse, 859–869
 - hyperbola, 870–884
 - overview, 846–847
 - parabola, 848–858
- Conjugate axis, of hyperbola, 871, 872, 877
- Conjugate Pairs Theorem, 353–356
- Conjugates, verifying trigonometric identities and, 569–570
- Connected mode, 246, 363
- Consistent system of linear equations
 - in three variables, 728
 - in two variables, 714
- Constant
 - defined, 2
 - of proportionality, 387
 - of variation, 387
- Constant function, 229–230, 240, 243
- Constant term, 30, 311
- Constraints, 757
- Contact force, 658
- Continuous compounding, 422–423
- Continuous curve, 312
- Conway, John, 196
- Coordinate, 4
- Coordinate line, 5, 170
 - distance between two points on, 8–9
- Coordinate plane, 170–172
 - algebraic vectors, 660–662
 - distance formula, 173–175
 - midpoint formula, 175
 - scales on graphing utility, 173
- Copernicus, Nicolaus, 637
- Cormack, Allan, 725
- Corner point, 750
- Correspondence diagram, 211–212
- Corresponding equation, 746
- Cosecant
 - cofunction identity for, 497
 - defined, 506
 - graphs of, 542–545
 - inverse, 551
 - properties of, 543
 - right triangle definition, 493
 - unit circle definition, 515
- Cosine. See also Law of Cosines
 - cofunction identity for, 497
 - defined, 506
 - double-angle identity for, 588
 - graphs of, 521–535
 - amplitude and period, 524–527
 - phase shift, 527–531
 - simple harmonic motion, 633–635
 - vertical shift, 531–533
 - half-angle identity for, 592
 - inverse, 549–553
 - period of, 521
 - power-reducing identity for, 590
 - product-to-sum identities and, 597–598
 - properties of, 523
 - right triangle definition, 493
 - sum and difference identities for, 574–576, 584
 - sum-to-product identities and, 599–600
 - unit circle definition of, 515
 - verifying trigonometric identities by converting to, 567–568, 570
- Cost
 - average, 222
 - fixed, 222
 - marginal, 222
 - variable, 222
- Cotangent
 - cofunction identity for, 497

- defined, 506
- graphs of, 538–542
- inverse, 551
- properties of, 540
- right triangle definition, 493
- unit circle definition, 515
- Coterminal angles, 480, 509
- Coughing, algebra in, 229, 231–232
- Coulomb's Law, 400
- Counting principles, 934–944
 - combinations, 938–940, 941–942
 - deciding which to use, 941–942
 - distinguishable permutations, 940–941
 - Fundamental Counting Principles, 934–936, 941–942
 - permutations, 936–938, 941–942
- Cramer, Gabriel, 832
- Cramer's Rule, 831–834
 - for solving three equations in three variables, 833–834
 - for solving two equations in two variables, 832
- Cricket chirps, conversion to degrees Celsius, 2, 15
- Cryptogram, 813
- Cryptography, 813, 821–823
- Cube root, 63
- Cube root function, 242–243, 244
- Cubes, difference and sum of, 36, 43–44
- Cubes (geometric), formulas for, 74
- Cubic equation, 351
- Cubic polynomials, 311
- Cubing function, 244
 - graphing, 233
- Current (I), 587
- Current ratio, 166
- Curves, sine, 522
- Cycle, 520
- Cyclotomic equation, 702

D

- Dantzig, George, 756
- Data, estimating probabilities using, 951–952
- Data matrix, 809
- Dating work of art, 444, 452–453
- da Vinci, Leonardo, 117
- Days, length of, 520, 532–533
- DD. See Decimal degree (DD) notation
- Decay, exponential, 424–425, 450
- Decimal degree (DD) notation, 481–482
- Decimal(s), 3–4
 - converting to scientific notation, 25
 - nonterminating repeating, 3
 - terminating, 3
- Decoding matrix, 823
- Decomposition
 - of functions, 272
 - of vector, 676–677
- Decreasing functions, 229–230
- Degenerate conic sections, 847
- Degree
 - of monomial, 36
 - of polynomial, 29, 30
- Degree, minute, second (DMS) notation, 482
- Degree measure, 480–481
 - minutes, seconds, and, 481
 - relation between radians and, 483–484
- DeHaven, Jacob, 417
- DeMoivre's n th Roots Theorem, 701
- DeMoivre's Theorem, 700–701
- Denominator, 12, 48. See also Least common denominator (LCD)
 - rationalizing, 62–63
- Dependent equations, 714, 716, 728, 729
- Dependent variable, 211
- Depreciation, linear, 199–200
- Depressed equation, 335
- Depression, angle of, 499
- Descartes, René, 170, 171
- Descartes's Rule of Signs, 343–344
- Descending order, 30
- Determinants, 827–837
 - cofactors, 828–829
 - Cramer's Rule, 831–834
 - history of, 827
 - of 2×2 matrix, 827–828
 - minors, 828–829
 - of $n \times n$ matrix, 830–831
- Dice, 891, 945–948
- Die galvanische Kette, mathematisch bearbeitet* (Ohm), 765
- Difference
 - of cubes
 - factoring, 43–44
 - products giving, 36
 - of functions, 268
 - of squares
 - factoring, 42–43
 - products giving, 36
- Difference identities
 - for cosine, 574–576, 584
 - for sine, 578–580, 584
 - for tangent, 583, 584
- Difference quotient, 235
- Digital calculator, 927
- Direct current (DC) circuits, 695
- Direction
 - of inequality, 148
 - vectors in terms of, 664–665
- Direction angle, 664–665
- Directly proportional, 387
- Directrix, of parabola, 849, 850, 853
- Direct variation, 386–388

- with powers, 388
- Discriminant, 125–126, 375
- Distance
 - absolute value and, 8
 - between cities, 486
 - of falling object, 903, 907
 - formula, 173–175
 - between two points, 8–9, 173–174
- Distinguishable permutations, 940–941
- Distributive Property, 12, 803, 808
- Dividend, 329, 738
- Division
 - complex conjugates and, 112–114
 - of complex numbers, 113–114
 - of fractions, 13–14
 - of polynomials, 328–339
 - long, 329–331
 - synthetic, 331–333
 - of rational expressions, 50
 - of real numbers, 12–14
- Division algorithm, 328–331
- Divisor, 329
- DMS. *See* Degree, minute, second (DMS) notation
- Domain
 - agreement on, 215
 - of algebraic expression, 15–16
 - of function, 215
 - composite, 271–272
 - linear, 240
 - logarithmic, 432
 - one-to-one, 284–285
 - rational, 360
 - from a graph, 218–219
 - of relation, 211
 - of variable
 - in equation, 83–84
 - in inequality, 147
- Dot mode, 246, 363, 539
- Dot product, 670–671
 - angle between two vectors, 671–673
 - decomposition of a vector, 676–677
 - orthogonal vectors, 673–674
 - projection of a vector, 674–676
 - properties, 672
 - work, 670, 677
- Double-angle identities, 587–590
- Double root, 120
- Doubling time, 468
- Dow Jones Industrial Average, 340
- Drugs
 - bioavailability of, 210, 220–221
 - cholesterol-reducing, 221
- Drug testing, false positive in, 48, 49
- DTMF. *See* Dual-tone multi-frequency (DTMF)
- Dual-tone multi-frequency (DTMF), 597, 601

- E
- Earth, radius of, 491
- Earthquakes, comparing magnitudes, 464
- Eccentricity
 - of ellipse, 869
 - of hyperbola, 884
- Economics
 - functions in, 222–223
 - Leontief input-output model, 784, 794–795
- Effective annual interest, 427
- Einstein, Albert, 132
- Electrical circuits
 - alternating current, 591–592, 695, 699–700
 - complex numbers and, 108, 114
 - direct variation in, 387–388
 - resistance in, 765, 773–774
- Electricity, 587
- Elementary row operations, 787
- Element(s)
 - matrix, 785
 - of set, 3
- Elevation, angle of, 499–501
- Elimination method
 - solving system of linear equations in two variables by, 717–718
 - solving systems of nonlinear equations by, 740–741
- Ellipse(s), 846, 847, 859–869
 - applications, 865–866
 - with center (0,0), 861
 - defined, 859
 - eccentricity of, 869
 - equation of, 860–862, 864–865
 - graphing, 862
 - horizontal, 861, 863
 - latus rectum of, 869
 - reflecting property of, 866
 - translations of, 863–865
 - vertical, 863
- Empty set, 6
- End behavior of polynomial functions, 313–316
- Endpoint, 479
 - showing inclusion/exclusion, 7
- Energy usage, measuring, 587, 591–592
- Entertainment industry, computer graphics and, 800
- Entry, matrix, 785
- Equality
 - of complex numbers, 110
 - of numbers, 3
 - of ordered pairs, 170
- Equally likely, 946
- Equal sets, 6
- Equal sign, 3
- Equations. *See also* Linear equations; Quadratic equations; Trigonometric equations

- of asymptotes of hyperbola, 874
- of circle, 185–188
- converting between rectangular and polar forms, 685–686
- cyclotomic, 702
- defined, 83
- depressed, 335
- of ellipse, 860–862, 864–865
- equivalent, 85
- exponential, 410, 456–459
- functions defined by, 213–214
- graphs of (*See under* Graph(s))
- of hyperbola, 871, 872, 873–874
- involving absolute value, 157–159
- of lines (*See* Lines, equations of)
- logarithmic, 461–464
- matrix, 803–804
- in one variable, 83
- of parabola, 849–853
- polar, 685–686
 - graphs of, 686–692
- quadratic in form, 139–42
- rational, 88–90
- solving, 84, 85
 - by factoring, 133–134
 - involving radicals, 136–138
 - in one variable, 85–87
 - quadratic in form, 139–142
 - rational, 134–136
 - with rational exponents, 138–139
- system of, 712–713
- Equilateral hyperbola, 884
- Equilateral triangle, 71, 495
- Equilibrium, 665, 666–667
- Equilibrium point, 719
- Equivalent equations, 85
- Equivalent inequalities, 147–148
- Equivalent systems, operations that produce, 727
- Equivalent vectors, 659
- Eratosthenes, 491
- ESWL. *See* Extracorporeal shock-wave lithotripsy (ESWL)
- Euclid, 846
- Euler, Leonhard, 109, 212, 421
- Euler number, 421
- Euler's Theorem, 825
- Even function, 232–233
- Even-odd identities, 565–566
- Event(s)
 - certain, 947
 - complement of, 950–951
 - defined, 946
 - impossible, 947
 - mutually exclusive, 949–950
 - probability of, 945–948
- Everest, George, 629
- Expanded form, of logarithmic expression, 446–447
- Expanding by cofactors, 830
- Experiment, 945
- Experimental probability, 951–952
- Exponential curve, matching data to, 450
- Exponential decay, 424–425, 450
- Exponential equations, 410
 - applications, 459–461
 - solving, 456–459
 - with different bases, 458
 - of quadratic form, 458–459
 - using logarithms, 458
- Exponential expressions, simplifying, 23–24
- Exponential form, 430
 - conversion with logarithmic form, 430–431
- Exponential functions, 405–406
 - compared to power functions, 412
 - evaluate, 406–407
 - graphing, 407–410
 - natural, 423–425
 - properties of, 409, 434
 - transformations on, 411–413
- Exponential growth, 424–425, 450
- Exponential inequalities, 465–466
- Exponents
 - integer, 19–21
 - power-of-a-product, 22–23
 - power-of-a-quotient, 23
 - Power Rule for, 22
 - Product Rule for, 21
 - Quotient Rule for, 21–22
 - radicals written as, 445
 - rational, 65–68
 - rules of, 21–23, 407
- Expressions. *See also* Algebraic expressions
 - exponential, simplifying, 23–24
- Extracorporeal shock-wave lithotripsy (ESWL), 859
- Extraneous solution (extraneous root), 88, 134, 135–136, 610–611, 619
- Extreme value, 231
- Exxon Valdez* oil spill, 267
- F
- Factored completely, 39
- Factorial
 - defined, 896
 - n , 896
 - zero, 896
- Factorial notation, 896–897
- Factoring
 - by grouping, 45–46
 - over integers, 39
 - polynomials
 - difference and sum of cubes, 43–44

- difference of squares, 42–43
- factoring out a monomial, 39–40
- formulas, 41
- greatest common monomial factor, 39
- by grouping, 45–46
- trinomials
 - of form $Ax^2 + Bx + C$, 44–45
 - of form $x^2 + bx + c$, 40–41
 - perfect-square, 42
- for quadratic equation, 118–120
- solving equations by, 133–134
- Factorization Theorem for Polynomials, 352
 - with real coefficients, 355
- Factors, 3, 39
 - polynomial, 328
- Factor Theorem, 335, 351
- Fahrenheit, conversion from Celsius, 90–91, 153
- Falling object, calculating distance of, 903, 907
- False positives in drug testing, 48, 49
- Family tree of honeybee, 892, 895–896
- Feasible solutions, 757
- Femur, inferring height from, 192, 199
- Fibonacci sequence, 895
- Finance, applied problems, 98–99
- Finite geometric sequence, sum of, 913–914
- Finite geometric series, 913–916
 - annuities, 914–916
- Finite sequence, 892
- Finite series, 899
- Fiore, Antonio Maria, 351
- First component, 170
- First component, of vector, 661
- First-degree equation. *See* Linear equations
- First row, of Pascal's triangle, 929
- Fixed cost, 222
- Flow velocity, average, 229, 231–232
- Focus (foci)
 - of ellipse, 859, 860, 861, 863
 - of hyperbola, 870, 872, 873, 877
 - of parabola, 849, 850, 853
- FOIL method, 33–34
 - for complex numbers, 111
- Foot-pounds, 670
- Force, 386, 658
 - resultant of system of, 658, 665
- Formula(s)
 - area, 74
 - change-of-base, 449
 - circumference, 74
 - combination, 939
 - compound interest, 420–423, 914
 - continuous, 422
 - defined, 90
 - distance, 173–175
 - on a number line, 8
 - factoring, 41
 - geometry, 74
 - half-life, 451
 - Heron's, 654–655
 - midpoint, 175
 - perimeter, 74
 - permutation, 937–938
 - quadratic, 124
 - recursive, 894–896
 - reduction, 581–582, 584
 - simple interest, 418
 - special-product, 36
 - temperature conversion, 90–91
 - volume, 74
- 45° - 45° - 90° triangle, 495
- Fraction bars, 9
- Fractions, 12
 - adding, 13–14
 - complex, 54–55
 - dividing, 13–14
 - multiplying, 13–14
 - partial, 766
 - properties of, 13
 - subtracting, 13–14
- Free-falling objects, 29, 31
- Frequency
 - beat, 602
 - of motion, 534
 - of wave, 574
- Friction, tap-water flow and, 59, 61–62
- Full moon, 613
- Functional notation, 212
- Function(s), 211–214
 - absolute value, 244, 245–246
 - applications, 220–221
 - average rate of change, 234–235
 - ceiling integer, 250
 - circular, 514, 516–517
 - combining, 267–269
 - composition of, 269–271
 - domain of, 271–272
 - constant, 229–230, 240, 243
 - cube root, 242–243, 244
 - cubing, 244
 - decomposition of, 272
 - decreasing, 229–230
 - defined by equations, 213–214
 - difference of, 268
 - domains of, 215
 - in economics, 222–223
 - even-odd, 232–233
 - exponential (*See* Exponential functions)
 - graphs of, 215–217
 - information from, 217–220
 - greatest integer, 247
 - hyperbolic cosine, 427
 - identity, 240, 243

increasing, 229–230
 inverse, 278–286
 linear, 240–242
 linear price-demand, 222
 logarithmic (See Logarithmic functions)
 periodic, 520–521
 piecewise, 244–245
 polynomial, 311–327
 power, 313
 product of, 268
 profit, 222
 properties of, 229–239
 quadratic, 300–310
 quotient of, 268
 rational (See Rational functions)
 reciprocal, 244, 361
 reciprocal square, 244
 relative maximum and minimum values, 230–232
 revenue, 222
 secant hyperbolic, 427
 square root, 242, 244
 squaring, 243
 step, 247
 sum of, 268
 tangent hyperbolic, 427
 transcendental, 424
 transformations (See *under* Transformations)
 Fundamental Counting Principle, 934–936, 941–942
 choosing, 941–942
 Fundamental identities, 497–499
 Fundamental rectangle, of hyperbola, 876
 Fundamental Theorem of Algebra, 352
 Fundamental trigonometric identities, 565
 Future value, 443
 of an annuity, 914

G

Galileo, 29, 31
 Gambling, 891
 Gauss, Karl Friedrich, 352, 906
 Gaussian elimination, 726–728, 789–792
 Gauss-Jordan elimination, 792–795, 815
 GCF. See Greatest common monomial factor (GCF)
 General form of an equation
 of a circle, 187
 of a line, 197–199
 General term
 of geometric sequence, 912
 of sequence, 892, 894
 Geometric definition of parabola, 849
 Geometric interpretation, of system of linear equations in three variables, 731–732

Geometric representation of complex numbers, 695–696
 Geometric sequences, 910–920
 annuities, 914–916
 common ratio of, 910–911
 definition of, 910
 finding sum of finite, 913–914
 general term of, 912
 infinite geometric series, 910, 916–917
 recursive definition of, 911
 Geometric series
 finite, 913–916
 infinite, 916–917
 Geometric vectors, 658–660
 adding, 659–660
 multiplying, 660
 subtracting, 659
 Geometry topics
 angles of triangle, 71
 applied problems, 97
 formulas, 74
 Pythagorean Theorem, 72–73
 converse of, 74
 proof of, 75
 triangles, 71–72
 Global positioning system (GPS), 870
 Global warming, 328
 Golden ratio, 117, 127–128
 Golden rectangle, 117, 127–128
 Graphical method, for system of linear equations in two variables, 713–714
 Graphing
 circle, 186
 combined vertical and horizontal shifts, 254–255
 compound inequality, 150–152
 cube root function, 243
 cubing function, 233
 ellipse, 861, 862, 863
 exponential functions, 407–410
 hyperbola
 with center (0,0), 872, 876–877
 with center (h,k), 878–880
 inequality using Reciprocal Sign Property, 152
 linear equation, 198–199
 linear inequalities, 148–149
 line using slope and y-intercept, 196
 logarithmic functions, 448
 Mach numbers, 544–545
 nonlinear inequality in two variables, 751
 parabola, 850–851, 853–854
 piecewise functions, 245–247
 points, 171
 polynomial function, 320–322
 quadratic function
 $f(x) = ax^2 + bx + c$, 303–305

- in standard form, 302–303
 - rational functions, 366–369
 - squaring function, 232
 - step function, 247
 - using reduction formula, 581–582
 - vertical shifts, 251–252
 - $y = a \cos bx$, 526–527
 - $y = a \cos b(x - c)$, 529–530
 - $y = a \cos [b(x - c)]$, 528–529
 - $y = a \cos b(x - c) + d$, 532
 - $y = a \cos x$, 524–525
 - $y = a \cot b(x - c)$, 540–541
 - $y = a \cot [b(x - c)]$, 541–542
 - $y = a \csc [b(x - c)]$, 543–544
 - $y = a \sin bx$, 527
 - $y = a \sin [b(x - c)]$, 528–529
 - $y = a \sin (bx - k)$, 531
 - $y = a \sin x$, 524
 - $y = a \sin (x - c)$, 527–528
 - $y = a \tan b(x - c)$, 540–541
 - $y = a \tan b(x - c)$, 540–541
 - of $y = \cot x$, 539–540
 - $y = \sin bx$, 525–526
- Graphing calculators
- absolute value function, 9
 - addition and subtraction of complex numbers, 110
 - addition of complex numbers, 110
 - binomial coefficients, 929
 - combinations, 939
 - complex conjugates, 112
 - complex numbers, 109
 - complex roots, 702
 - composite function, 270
 - connected mode, 246
 - converting between degrees and radians, 484
 - converting between polar and rectangular forms, 683, 684
 - determinants, 830
 - division of complex numbers, 114
 - dot mode, 246
 - elementary row operations, 788
 - equation of a circle, 186
 - evaluation of function, 214
 - exponential functions, 406, 409
 - graphing ellipse, 862
 - graphing trigonometric functions, 539
 - graphs of polar equations, 688
 - greatest integer function, 246
 - horizontal shifts, 253, 260, 262
 - integer exponents, 20
 - intersect feature, 714
 - inverse function, 285
 - matrices, 787
 - matrix addition, 802
 - matrix inverse, 815
 - matrix multiplication, 807
 - matrix subtraction, 803
 - order of operations, 10
 - parabola, 851, 852
 - partial-fraction decomposition, 769
 - permutations, 938
 - powers of complex numbers, 701
 - quadratic inequality, 377
 - rational functions, 363
 - real zeros, 346
 - reduced row-echelon form, 792
 - revenue curve, 371
 - scientific notation, 25
 - sequences, 893
 - shading graphs, 748
 - sinusoidal graphs, 526, 530
 - square roots, 60
 - subtraction of complex numbers, 110
 - sum of finite geometric sequences, 914
 - sum of terms of series, 898
 - synthetic division, 331
 - systems of linear inequalities in two variables, 749
 - systems of nonlinear equations, 740
 - trigonometric equations, 609, 610, 614, 615
 - trigonometric identities, 566, 567, 571, 576
 - vertical shifts, 252, 260, 262
- Graphing utility
- linear regression, 202–203
 - scales on, 173
- Graph(s), 4
- bar, 173
 - of cosecant function, 542–545
 - of cosine function, 521–535
 - of cotangent function, 538–542
 - of equations, 179–181
 - circles, 185–188
 - intercepts, 181
 - symmetry, 182–185
 - of functions, 212, 215–217
 - logarithmic, 433–435
 - quadratic, 301
 - line, 173
 - of linear inequalities in two variables, 745–748
 - obtaining information from, 219–220
 - of ordered pair, 171
 - of polar equation, 686–692
 - of reflections, 255–256
 - of revenue curve, 370–371
 - of secant functions, 542–545
 - of sine function, 521–535
 - amplitude and period, 524–527
 - phase shift, 527–531
 - simple harmonic motion, 633–635
 - sinusoidal, 521–535
 - amplitude and period, 524–527
 - phase shift, 527–531
 - simple harmonic motion, 633–635

- vertical shift, 531–533
- of tangent function, 538–542
- Gravitation, Law of Universal, 386, 391
- Greater than*, 5, 7
- Greater than or equal to*, 5, 7
- Greatest common monomial factor (GCF), 39
- Greatest integer function, 247
- Great Trigonometric Survey (GTS), 629
- Greenhouse effect, 328
- Grid cells, 725, 732–733
- Gross production matrix, 820
- Grouping
 - factoring by, 45–46
 - symbols, 9
- Group number, in Social Security number, 934
- Growth
 - exponential, 424–425, 450
 - population, 460–461
- Growth factor, 405
- Growth rate, 405, 413
- GSP. *See* Global positioning system (GPS)

H

- Hales, Stephen, 251
- Half-angle identities, 592–594
- Half-life, 450–451
- Half moon, 613
- Half plane, 746
- Halley, Edmund, 865, 866, 868
- Halley's comet, 865–866, 868
- Harmonic motion, simple, 533–535
- Harmonic sequence, 909
- Hawass, Zahi, 444
- Height, estimating
 - from femur, 192, 199
 - of mountain, 632–633
 - of Mount Everest, 629
 - of Mount Kilimanjaro, 492, 500–501
 - using angle of elevation, 499–501
 - using reference angles, 513–514
- Heron's formula, 654–655
- Hertz, Gustav, 574
- Hertz (Hz), 574
- Hipparchus, 564
- Honeybee family tree, 892, 895–896
- Hooke's Law, 393
- Horizontal asymptotes, of rational function, 364–366
- Horizontal ellipse, 861, 863
- Horizontal lines
 - equation of, 196–197, 203
 - slope of, 194
- Horizontal-line test, 279–280
- Horizontal method
 - of adding polynomials, 31
 - of multiplying polynomials, 32

- of subtracting polynomials, 32
- Horizontal shifts, 252–255, 260, 262
 - on exponential functions, 411
- Horizontal stretch and compression, 258–259, 262
 - on exponential functions, 411
- Horsepower, 745
- Hounsfield, Godfrey, 725
- Hounsfield numbers, 732
- Hubble, Edwin Powell, 393, 848
- Hubble Space Telescope, 848, 855
- Humphries, Robert, 945
- Hyperbola(s), 846, 847, 870–884
 - applications, 880–881
 - asymptotes of, 874–875
 - centered
 - at (0,0), 872
 - at (h,k), 877
 - defined, 870–874
 - eccentricity of, 884
 - equations of, 871, 872, 873–874
 - equilateral, 884
 - graphing
 - with center (0,0), 876–877
 - with center (h,k), 878–880
 - latus rectum of, 884
 - LORAN, 870, 880–881
 - orientation of, 872
 - parts of, 871
 - standard position of, 870, 871
 - translations of, 877–880
- Hyperbolic cosine function, 427
- Hyperbolic mirror, 880
- Hypersonic speed, 538
- Hypotenuse, 72, 492

I

- i, j** form, vectors in, 663
- Identity, 84, 87. *See also* Trigonometric identities
 - defined, 564
 - inequality, 147
- Identity function, 240, 243
- Identity matrix, 808
- Identity Properties, 10, 12
 - (i,j)th entry, 785
- Imaginary axis, 695
- Imaginary number, pure, 109
- Imaginary part, of complex number, 108, 109
- Imaginary unit, 108
- Impedance, 108, 695
- Impossible event, 947
- Improper, 329
- Improper rational expression, 766
- Included angle, 72, 629
- Included side, 72, 629
- Inconsistent equation, 84, 87
- Inconsistent inequality, 147

Inconsistent rational equation, 89–90
 Inconsistent system of linear equations, 792
 in three variables, 728
 in two variables, 714, 715–716
 Increasing functions, 229–230
 Independent equations, 714, 728
 Independent variable, 211
 Index(es), 63
 radicals with different, 65
 of summation, 898
 Inductive definition, 830
 Inequalities, 5, 146–148
 compound, 150–152
 conditional, 147
 equivalent, 147–148
 exponential, 465–466
 identifying, 5
 inconsistent, 147
 involving absolute value, 159–161
 linear (See Linear inequalities)
 logarithmic, 465–466
 polynomial, 376–381
 properties of, 5
 rational, 381–383
 systems of, 745–755
 Inequality symbols, 5, 7
 Infinite geometric series, 910, 916–917
 Infinite sequence, 892
 Infinite series, 899
 Infinity symbol, 7
 Initial point, of vector, 658
 Inscribed circle, 657
 Integer exponents, 19–21
 Integers, 3, 4
 factoring over the, 39
 Intercepts, 181
 sketching graphs using, 184–185
 x -, 181
 y -, 181, 196
 Interest, 98, 417
 compound, 418–421
 simple, 98–99, 417–418
 Interest rate, 98, 417, 459
 effective annual, 427
 Interindustry technology input-output matrix, 820
 Intermediate Value Theorem, 317–318, 341
 Intersecting lines, in degenerate conic section, 847
 Intersection of sets, 6
 Interval notation, 7
 Interval of values, for linear expression, 153
 Intervals, 6–7
 Inverse, 11, 12
 matrix (See Matrix inverse)
 Inverse Function Property, 281
 Inverse functions, 278–282
 applications, 286
 of f , 280
 finding, 282–284
 range of one-to-one function, 284–285
 graphing logarithmic functions using, 433–434
 relating values of function and its inverse, 280–281
 Inversely proportional, 389
 Inverse trigonometric functions, 547–556
 composition of trigonometric functions and, 553–555
 inverse cosecant, 551
 inverse cosine, 549, 551, 553
 inverse cotangent, 551
 inverse secant, 551
 inverse sine, 547–548, 551, 553
 inverse tangent, 550, 551, 553, 618–619
 solving trigonometric equations containing, 618–620
 using calculator with, 551–553
 Inverse variation, 389–390
 with powers, 389–390
 Invertible matrix, 815
 Investments
 analysis of, 98
 doubling your money, 437–438
 return on, 39, 43–44
 stock market, 738, 741–742
 Irrational numbers, 3–4
 Irreducible polynomials, 39, 42
 Isosceles right triangle, 495
 Isosceles triangle, 71
 Iteration, 926

J
 Jacobi, Carl Gustav Jacob, 827
 Joint and combined variation, 390–392

K
 Kelvin temperature scale, 288
 Kepler, Johannes, 311, 322, 435, 865, 868
 Kepler's Third Law, 395
 Key points, of cycle of graph, 523
 Kilowatt hours, 587
 King Tut's treasure, 444, 452–453
Kitab al-jabr wal-muqabala, 712
 Koch's snowflake, 925
 Köwa, Seki, 827
 Kremp, Christian, 896
 Kronecker, Leopold, 827

L
 Lambton, William, 629
 Latitude, 479, 486
 Latus rectum
 of ellipse, 869
 of hyperbola, 884

- of parabola, 852
- Law of Cooling, 438–440
- Law of Cosines, 644
 - derivation of, 645
 - dot product and, 671
 - solving SAS triangles, 645–647
 - solving SSS triangles, 647–648
- Law of Sines, 629–636, 630–631
 - ambiguous cases, 637–643
- Law of Universal Gravitation, 386, 391
- LCD. *See* Least common denominator (LCD)
- Leading coefficient, 30, 311
- Leading entry, of matrix, 789
- Leading term, 30, 311
- Leading-term test, 315–316
- Least common denominator (LCD), 51
 - finding for rational expressions, 52–53
 - using to add and subtract rational expressions, 53–54
- Least-squares lines, 201
- Legs, of triangle, 72, 492
- Leibniz, Gottfried, 827
- Lemniscates, 692
- Length
 - ABC formula, 486
 - of arc of circle, 485–486
 - of days, 520, 532–533
 - determining, 240, 241–242
- Leonardo of Pisa (Fibonacci), 895
- Leontief, Wassily, 784
- Leontief input-output model, 784, 794–795, 820–821
- Less than*, 5, 7
- Less than or equal to*, 5, 7
- Libby, Williard Frank, 452
- Liebeck, Stella, 429
- Lie in a quadrant*, 480
- Light, speed of, 390
- Like radicals, 64–65
- Like terms, 30
- Limaçon, 690–692
- Linear attenuation units (LAUs), 732
- Linear depreciation, 199–200
- Linear equation in the variables x_1, x_2, \dots, x_n , 725–726
- Linear equations. *See also under* Systems of linear equations
 - applications, 96–107
 - conditional, 84, 87
 - defined, 192
 - identity, 84, 87
 - inconsistent, 84, 87
 - in one variable, 83–95
 - applications, 91
 - solving, 85–87
 - solving, 85
 - types of, 87
- Linear expressions, inequalities involving
 - reciprocal of, 152–153
- Linear functions, 240–242
- Linear growth, 418
- Linear inequalities, 148–156
 - combining two, 150–152
 - involving reciprocal of linear expression, 152–153
 - in two variables, graph of, 745–748
- Linear price-demand function, 222
- Linear programming, 756–757
 - applications, 760–761
 - solving problems, 757–760
- Linear regression, modeling data using, 201–203
- Linear speed, 487–488
- Linear system, 726
 - of equations (*See under* Systems of linear equations)
- Linear trigonometric equation, 607
- Line graph, 173
- Line of symmetry, 182
- Lines, 192–209
 - equations of, 194–196
 - general form, 197–199
 - horizontal lines, 196–197, 203
 - parallel lines, 200–201
 - perpendicular lines, 200–201
 - point-slope, 194–195, 203
 - slope-intercept, 196, 203
 - vertical lines, 196–197, 203
 - least-squares, 201
 - linear depreciation, 199–200
 - modeling data using linear regression, 201–203
 - parallel, 200–201, 203
 - perpendicular, 200–201, 203
 - regression, 201
 - slope of, 192–194
- Line segment, finding midpoint of, 175
- Lithotripsy, 859, 866
- Lithotripter, 859, 866
- Loan repayment, 417, 422–423
- Logarithmic equations, solving, 461–464
 - in logistic growth model, 463–464
 - using one-to-one property of logarithms, 462
 - using Product and Quotient Rules, 462–463
- Logarithmic form, 430
 - conversion with exponential form, 430–431
- Logarithmic functions, 429–431
 - domain of, 432
 - graphs of, 433–435
 - investments, 437–438
 - natural, 437
 - Newton's Law of Cooling, 438–440
 - properties of, 434
- Logarithmic inequalities, 465–466

- Logarithmic tables, 435
- Logarithms
 - change of base, 449–450
 - common, 435–436
 - evaluating, 431–432
 - growth and decay, 450
 - half-life, 450–451
 - natural, 437
 - properties of, 433, 437
 - radiocarbon dating, 452–453
 - rules of, 444–448
 - solving exponential equations using, 458
- Logistic curve, 456
- Logistic growth model, 456, 463–464
- London Eye, 506, 513–514
- Long division, 329–331
- Longitude, 479, 486
- Longley, Wild Bill, 192
- Loomis, Alfred Lee, 870
- LORAN (LONg-RANge Navigation), 870, 880–881
- LORAN (LONg-RANge Navigation)-C, 870
- Loss, taking, 222
- Lotteries, 945, 948
- Lower bound, 344
- Lower limit, of summation, 898
- Lowest terms, 361

- M
- Mach numbers, 538
 - graphing, 544–545
- Magnitude
 - of complex number, 696, 697
 - of sum of two vectors, 673
 - of vector, 658
 - vectors in terms of, 664–665
- Main diagonal, of matrix, 785
- Maisey, John, 240
- Major axis, of ellipse, 859, 860, 861, 863
- Mallows, Colin L., 902
- Malthus, Thomas Robert, 393
- Marginal cost, 222
- Mathematical car, 800
- Mathematical induction, 921–926
 - determining statement P_{k+1} from statement P_k , 922–924
 - principle of, 922
- Mathematical modeling, 96, 784
- Mathematics, in the movies, 71
- Matrix algebra, 800–812
 - computer graphics, 800, 808–809
 - equality of matrices, 800–801
 - matrix addition, 801–802
 - matrix multiplication, 804–808
 - matrix subtraction, 802–803
 - scalar multiplication, 802–804
- Matrix equation, 803–804
- Matrix inverse, 813–826
 - applications, 820–821
 - cryptology, 813, 821–823
 - finding, 815–817
 - inverse of 2×2 matrix, 818
 - multiplicative, 813–814
 - proving nonzero matrix has no inverse, 814
 - solving systems of linear equations using, 818–820
- Matrix (matrices)
 - augmented, 785–786, 788
 - coding, 822
 - coefficient, 785–786
 - column, 785
 - data, 809
 - decoding, 823
 - defined, 784–785
 - equality of, 800–801
 - Gaussian elimination, 789–792
 - Gauss-Jordan elimination, 792–795
 - gross production, 820
 - identity, 808
 - interindustry technology input-output (technology), 820
 - invertible, 815
 - negative, 803
 - nonsingular, 815
 - order of, 785
 - reduced row-echelon form, 789
 - row, 785
 - row-echelon form, 789
 - row-equivalent, 787
 - solving system of linear equations using, 785–789
 - zero, 803
- Matrix multiplication, 804–808
 - defining product AB , 804–805
 - product of $1 \times n$ and $n \times 1$ matrices, 805–806
 - properties of, 808
- Maurolico, Francesco, 923
- Maximum value
 - of function, 756
 - of parabola, 301
- McDonald's hot coffee, 429, 438–439
- Mean, arithmetic, 909
- Megatooth shark, 240, 241–242
- Members, of set, 3
- Mersenne, Marin, 927
- Midpoint formula, 175
- Military, computer graphics and, 800
- Millennium Wheel, 506, 513–514
- Minimum value
 - of function, 756
 - of parabola, 301
- Minor axis, of ellipse, 859, 860, 861, 863
- Minors, 828–829

Minutes, 481
 Mixture problems, 103
 Model for exponential growth or decay, 424–425
 Modulus, 696, 697
Mona Lisa (da Vinci), 117
 Monomial, 29–30, 36
 factoring out, 39–40
 Moon, phases of, 613, 615–616
 Motion
 projectile, 604, 608
 simple harmonic, 533–535
 Mountain, estimating height of, 632–633
 Mount Everest, estimating height of, 629
 Mount Kilimanjaro, estimating height of, 492, 500–501
 Multiple angle, 614
 Multiplier, 910
 Multiplier effect, 910
 Multiplication
 of complex numbers, 111–112
 of fractions, 13–14
 matrix, 804–808
 of polynomials, 32–33, 36
 of radicals with different indexes, 65
 of rational expressions, 50
 scalar, 802–804
 of vector, 660
 Multiplicative identity, 10
 Multiplicative inverse, 813–814
 Multiplicative Inverse Property, 11
 Multiplicity of a zero, 319
 Multiplier effect, 917
 Music, pure tones, 574, 582
 Mutually exclusive events, 949–950

N
 Napier, John, 435
 Nappes of the cone, 846
 Natural exponential function, 423–425
 Natural logarithms, 437
 Natural numbers, 3, 4
 mathematical induction and, 921–922
 Navigation
 using bearings, 633–634
 vectors in air, 666
 Navigational systems, 880–881
n choose *r*
 defined, 929
 evaluating, 929–930
 Negative angle, 480
 Negative infinity symbol, 7
 Negative integer exponents, 20
 Negative matrix, 803
 Negative numbers, 4
 square roots of, 109
 multiplication involving, 112

New moon, 613
 Newton, Isaac, 386, 391, 865
 Newton's Law of Cooling, 438–440
 Newtons (N), 658
n factorial, 896
n × *n* matrix
 determinant of, 830–831
 minors and cofactors of, 828–829
 No degree, 29
 Nominal rate, 419
 Nonlinear inequality, 751
 Nonlinear systems of inequalities, 752–753
 Nonnegative identity, 147
 Nonrigid transformations, 257
 Nonsingular matrix, 815
 Nonsquare linear systems, 731
 Nonterminating repeating decimal, 3
 Nonzero row, 789
 Norm, of vector, 658
 Notation
 angle, 481–482
 decimal degree, 481–482
 degree, minute, second (DMS), 482
 factorial, 896–897
 functional, 212
 interval, 7
 scientific, 24–26
 set-builder, 5–6, 7
 summation (sigma), 898–899
Nova Stereometria Doliorum Vinariorum, 311
*n*th partial sum, 899
*n*th roots, 63–64
 product and quotient rules for, 64
 of unity, 702–703
*n*th row, of Pascal's triangle, 929
*n*th term
 of arithmetic sequence, 904–905
 of geometric sequence, 912
 of sequence, 892
 Null set, 6
 Number line, 5, 170
 distance between two points on, 8–9
 Number of Zeros Theorem, 353
 Number(s)
 classifying, 2–3
 sets of, 3
 complex, 108–116
 equality of, 3
 Euler, 421
 irrational, 3–4
 Mach, 538, 544–545
 natural, 3, 4, 688–689
 negative, 4
 positive, 4
 rational, 3–4
 real, 4

transcendental, 424
whole, 3, 4
Numerator, 12, 48
Nutrition, 760–761

O

Objective function, 757
Oblique asymptotes, of rational function, 369–370
Oblique triangles, solving, 629–630
Obtuse angle, 71
Odd-degree polynomials with real zeros, 354
Odd function, 233
Ohm, Georg Simon, 765
Ohm's Law, 765
Oil spill, calculating area of, 272–273
One degree measure, 480
1 radian, 482
One-Sign Theorem, 379
One-to-one function, 278–282, 429
 applications, 286
 finding, 282–284
 range of, 284–285
One-to-One Property of Logarithms, 462
Open interval, 7
Opposite number, 11
Opposites, properties of, 13
Opposite vector, 659
Optimal solution, 757
Optimization, 756–757
Orbital period, 395
Ordered, 5
Ordered pair, 170
 graph of, 171
Order m by n , 785
Order of operations, 10
Origin
 of coordinate plane, 170
 of number line, 5
 symmetric with respect to, 182, 184
Orthogonal vectors, 673–674

P

Parabola(s), 846, 847, 848–858
 equation of, 849–853
 geometric definition of, 849
 latus rectum, 852
 reflecting property of, 854–855
 translations of, 853–854
 with vertex $(0,0)$ and focus $(a,0)$, 850
 with vertex (h,k) and a **greater than 0**, 853
Parallel circuit, 765
Parallel lines, 200–201, 203
Parallel property of resistors, 765
Parallel vectors, 671–672
Parentheses, as grouping symbols, 9
Parthenon, 117

Partial fraction, 766
Partial-fraction decomposition, 765–775
 partial fractions, 766
 $Q(x)$
 has distinct irreducible quadratic factors, 770–771
 has only distinct linear factors, 767–769
 has repeated irreducible quadratic factors, 771–774
 has repeated linear factors, 769–770
Partial sums, 905–907
 writing in summation notation, 899–900
Pascal, Blaise, 927, 928
Pascaline, 927
Pascal's triangle, 927, 928–932
Pasteur, Louis, 413
Pasteurization, 413
Pattern method, of squaring a binomial sum, 34
Peak, 220
PEMDAS, 10
Pendulum, period of, 288, 394
Perfect squares, 4
 finding square roots of, 60
Perfect-square trinomials, 121
 factoring, 42
Perfect triangles, 656
Perihelion, 868
Perimeter formulas, 74
Period, 520–521, 524–527
 finding, 531
 of pendulum, 288, 394
 of sinusoidal graph/curve, 524–527
Periodic functions, 520–521
Periodic motion, 394
Permutations, 936–938
 choosing, 941
 defined, 936
 distinguishable, 940–941
 of n objects
 number of, 937
 taken r at a time, 937
Perpendicular lines, 200–201, 203
Petroleum consumption, 336
Pharmacokinetics, 210, 220–221
Phase shift, 527–531
Phases of the moon, 613, 615–616
Piecewise functions, 244–245
 graphing, 245–247
Pitiscus, 492
Plane, complex, 695
Planetary orbits, 637, 865, 868
Playing cards, 949–950
Plotting points
 for complex numbers, 695–696
 graphing logarithmic functions using, 433–434
 sketching graph by, 180

- Point(s), 847
 - on a graph, 218
 - graphing, 171
 - plotting, 180, 433–434, 695–696
- Point-slope form, of equation of a line, 194–195, 203
- Poiseuille, Jean Louis Marie, 251
- Poiseuille's Law, 251, 261
- Polar axis, 680
- Polar coordinates, 680–682
 - converting between polar and rectangular forms, 682–685
 - converting equations between rectangular and polar forms, 685–686
 - graph of polar equation, 686–692
 - tests for symmetry in, 687
- Polar coordinate system, 680
- Polar equations, 685–686
 - graphs of, 686–692
- Polar form of complex numbers, 696–698
 - powers of, 700–701
 - product and quotient in, 698–700
 - roots of, 701–703
- Polynomial factor, 328
- Polynomial functions, 311–313
 - common properties, 312
 - complex zeros of, 351–358
 - of degree n , 311
 - end behavior of, 313–316
 - graphing, 320–322
 - power functions, 313
 - real zeros of, 316–318, 340–350, 354
 - zeros and turning points, 318–320
- Polynomial inequalities, 376
 - using test points to solve, 376–381
- Polynomials, 29–38
 - adding, 31
 - complex, 351–352
 - cubic, 311
 - dividing, 328–339
 - applications, 336
 - division algorithm, 328–331
 - Factor Theorem, 335
 - Remainder Theorem, 333–334
 - synthetic division, 331–333
 - in factored form, 341
 - factoring (*See under* Factoring)
 - Factorization Theorem for, 352
 - irreducible, 42
 - multiplying, 32–33, 36
 - odd-degree, with real zeros, 354
 - in one variable, 30
 - product of the sum and difference of terms, 35–36
 - quartic, 311
 - quintic, 311
 - special products, 3–34
 - squaring a binomial sum or difference, 34–35
 - standard form, 30
 - subtracting, 31–32
 - vocabulary, 29–31
 - zero, 30
- Population dynamics, 179
- Population growth, 460–461
- Position vector, 660–661
- Positive angle, 480
- Positive integer exponent, 19
- Positive numbers, 4
- Pounds (lb), 658
- Power functions, 313
 - compared to exponential functions, 412
 - of degree n , 313
 - of even degree, 313
 - of odd degree, 313
- Power-of-a-Product Rule, 22–23
- Power-of-a-Quotient Rules, 23
- Power (P), 587
- Power-reducing identities, 590–591
- Power Rule
 - for exponents, 22
 - for logarithms, 445, 446
- Power(s)
 - of complex numbers, in polar form, 700–701
 - direct variation with, 388
 - inverse variation with, 389–390
- Prime meridian, 479
- Principal, 98, 417
- Principal square root, 59
- Principle of Mathematical Induction, 922
- Probability, 891, 927, 945–955
 - Additive Rule, 948–949
 - of an event, 945–948
 - complement of an event, 950–951
 - experimental, 951–952
 - mutually exclusive events, 949–950
 - theoretical, 951
- Product
 - of functions, 268
 - giving a difference of squares, 35–36
 - in polar form, 698–700
 - scalar, 802
- Product AB , 804–808
- Product Property of Square Roots, 61
- Product Rule
 - for exponents, 21
 - for logarithms, 445, 446
 - for n th roots, 64
 - solving logarithmic equations using, 462–463
- Product-to-sum identities, 597–598
- Profit
 - determining, from research and development, 39, 43–44

- earning, 222
- Profit function, 222
- Programming, 756
- Projectile motion, 604, 608
- Projection, of vector, 674–676
- Proper, 329
- Proper rational expression, 766
- Proportionality constant, 387
- Pure imaginary number, 109
- Pure tones, 574, 582
- Pyramid of Giza, 117
- Pythagorean identities, 498, 565, 610
- Pythagorean Theorem, 71, 72–73, 322, 644
 - converse of, 74
 - finding trigonometric function values using, 495–496
 - proof of, 75

Q

- Quadrantal angles, 508–509
- Quadrants, 170
- Quadratic equations
 - applications, 126–127
 - completing the square, 121–123
 - with complex solutions, 125
 - defined, 118
 - discriminant, 125–126
 - factoring method for, 118–120
 - golden rectangle, 117, 127–128
 - quadratic formula, 123–125
 - square root method for, 120
- Quadratic formula, 123–125
- Quadratic functions, 300–301
 - applications, 305
 - finding, using graph points, 733–734
 - $f(x) = ax^2 + bx + c$, 303–305
 - standard form, 301–303
- Quadratic inequality
 - using One-Sign Theorem to solve, 379
 - using test-point method to solve, 377
- Quadratic in form, solving equations that are, 139–142
- Quadratic trigonometric equation, 609
- Quartic polynomials, 311
- Queen bee, 892
- Queen Elizabeth 2*, 96, 101
- Quetelet, Lambert, 464
- Quintic polynomials, 311
- Quotient, 12, 329
 - difference, 235
 - of functions, 268
 - in polar form, 698–700
- Quotient identities, 498, 565
- Quotient Property of Square Roots, 61
- Quotient Rule
 - for exponents, 21–22

- for logarithms, 445, 446
- for n th roots, 64
- solving logarithmic equations using, 462–463

R

- Radian measure, 482–483
- Radians, 480
 - relation between degrees and, 483–484
- Radical expressions, 63
 - addition of, 64–65
 - subtraction of, 64–65
- Radicals, 60. *See also* Square roots
 - cube roots, 63
 - with different indexes, 65
 - like, 64–65
 - n th roots, 63–64
 - rational exponents and, 66, 68
 - solving equations involving, 136–138
 - written as exponents, 445
- Radical sign, 59, 63
- Radicand, 60, 63
- Radiocarbon dating, 452–453
- Radius of circle, 185
- Randomly selected, 952
- Range
 - from a graph, 218–219
 - of logarithmic function, 432
 - of one-to-one function, 284–285
 - of relation, 211
- Rate of work, 101–103
- Ratio, 12
- Rational approximation, 4
- Rational equations, 88–90
 - solving, 134–136
- Rational exponents, 65–68
 - properties of, 67
 - solving equations with, 138–139
- Rational expressions, 48–49
 - addition of, 51–54
 - complex, 54–55
 - complex fractions, 54–55
 - division of, 50
 - lowest terms for, 49
 - multiplication of, 50
 - subtraction of, 51–54
- Rational functions, 359–361
 - graphing, 366–369
 - graph of revenue curve, 359, 370–371
 - horizontal asymptotes, 364–366
 - oblique asymptotes, 369–370
 - vertical asymptotes, 361–364
 - whose graph has a hole, 363–364
- Rational inequalities, 381–383
- Rationalizing denominators, 62–63
- Rational numbers, 3–4
- Rational Zeros Theorem, 341–343

Ray, 479
 Real axis, 695
 Real number line (real line), 4–5
 distance between two points on, 8–9
 Real numbers
 absolute value, 8
 additive inverse, 11
 algebraic expressions, 14–16
 arithmetic expressions, 9–10
 complex numbers and, 108
 defined, 4
 division of, 12–14
 inequalities, 5
 intervals, 6–7
 properties of, 10–11, 12
 reciprocals, 11–12
 relationships among sets of, 4
 subtraction of, 12–14
 trigonometric functions of, 514–515
 Real part, of complex number, 108, 109
 Real zeros of polynomial functions, 316–319, 340–350. *See also* Complex zeros of polynomial functions
 bounds on real zeros, 344–345
 defined, 340
 Descartes's Rule of Signs, 343–344
 finding, 345–347
 odd-degree polynomials with, 354
 Rational Zeros Theorem, 341–343
 Reciprocal, 11–12
 negative exponents and, 20
 Reciprocal function, 244, 361
 Reciprocal identities, 498, 565
 Reciprocal Sign Property, 152
 Reciprocal square function, 244
 Rectangle(s)
 formulas for, 74
 fundamental, 876
 Rectangular coordinates
 converting from polar to, 682–683
 converting to polar coordinates, 683–684
 Rectangular form, of complex number, 696, 698
 Rectangular solid, formulas for, 74
 Recursive definition
 of arithmetic sequence, 904
 of geometric sequence, 911
 Recursive formulas, 894–896
 Reduced row-echelon form matrix, 789
 Gauss-Jordan elimination, 792–795
 Reduction formula, 581–582, 584
 Reference angles, 510–511
 using, 511–514
 Reflecting property
 of ellipses, 866
 of hyperbolas, 880
 of parabolas, 854–855
 Reflecting telescope, 880
 Reflections, 255–256, 260, 262
 on exponential functions, 411
 Regression line, 201
 Relations, 179, 210–211
 Relative Frequency Principle, 951
 Relative maximum value, 230–232
 Relative minimum value, 230–232
 Remainder, 329
 Remainder Theorem, 333–334
 Research and development, determining profit
 from, 39, 43–44
 Resistance, calculating in a circuit, 765, 773–774
 Resultant, of system of force, 658, 665
 Resultant vector, 659
 Retail theft/security, 547, 555
 Return, on investment, 39, 43–44
 Revenue, calculating sales, 805
 Revenue curve, 359, 370–371
 Revenue function, 222
 Revolutions per minute, angular and linear speed
 from, 487–488
 Richter scale, 464
 Right angle, 480
 Right angle trigonometry, 492–505
 applications, 499–501
 cofunction identities, 497
 complements, 497
 function values for special angles, 495–496
 fundamental identities, 497–501
 trigonometric ratios and functions, 492–495
 Right circular cone, 846
 formulas for, 74
 Right circular cylinder formulas, 74
 Right triangle(s), 72, 173
 identifying, 174
 isosceles, 495
 Rigid transformations, 257
 Rise, 192
 River, finding width of, 501
 Robotic hand, positioning, 684–685
 Roosevelt, Franklin D., 870
 Root of multiplicity of 2, 120
 Roots, 63–64. *See also* Square roots
 of complex numbers, 701–703
 cube, 63
 of equation, 84
 principal n th roots, 63–54
 Rose curves, 692
 Roster method, 5
 Rotation angle, 555
 Row-echelon form matrix, 789
 Gaussian elimination, 789–792
 reduced, 789
 Row-equivalent matrices, 787
 Row matrix, 785

Rule of 70, 469
 Run, 192

S
 Sales, applying composition to, 273–274
 Sample space, 945
 SAS (side-angle-side) Theorem, 72
 SAS triangles, 629, 630
 area of, 651–653
 Law of Cosines and, 644
 solving, 645–647
 Satisfy an equation, 84, 179
 Scalar, 658, 802
 dot product of two vectors, 670
 Scalar identity property, 803
 Scalar multiple, 660
 Scalar multiplication, 802–804
 associative property of, 808
 of vectors, 660, 662
 Scalar product, 802
 Scalar projection of v onto w , 674–676
 Scalar quantities, 658
 Scalene triangle, 71
 Scales on a graphing utility, 173
 Scatter diagram, 172, 212
 Scientific notation, 24–26
 converting decimal number to, 25
 Scott, David R., 29
 Search range of aircraft, 157, 160–161
 Secant
 cofunction identity for, 497
 defined, 506
 graphs of, 542–545
 inverse, 551
 properties of, 543
 right triangle definition, 493
 unit circle definition, 515
 Secant hyperbolic function, 427
 Second component, of vector, 661
 Seconds, 481
 Sector, area of, 488
 Semiperimeter, 654
 Sense, of inequality, 148
 Sequences, 892–899. *See also* Arithmetic sequences (arithmetic progressions); Geometric sequences
 defined, 892
 factorial notation, 896–897
 Fibonacci, 895
 finite, 892
 harmonic, 909
 infinite, 892
 recursive formulas, 894–896
 series, 899–900
 summation notation, 898–899
 writing terms of, 893–894, 897

Serial number, in Social Security number, 934
 Series, 899–900
 finite, 899
 finite geometric, 913–916
 infinite, 899
 infinite geometric, 910, 916–917
 Set, 3, 5–6
 empty (null), 6
 of feasible solutions, 757
 finding complement of, 6
 solution, 84
 Set-builder notation, 5–6, 7
 Set notation, 945
 Shareholder, 738
 Sickdhar, Radhanath, 629
 Sierpinski's triangle, 925–926
 Sigma notation, 898–899
 writing partial sum in, 899–900
 Signs
 Descartes's Rule of, 343–344
 of trigonometric functions, 509–510
 Similar triangles, 72, 493, 629
 Simple harmonic motion, 533–535
 of pure tone, 574
 Simple interest, 98–99, 417–418
 Simplifying
 complex fractions, 54–55
 exponential expressions, 23–24
 radicals, 68
 square roots, 61–62
Simpsons, The, 72
 Simultaneous operations, 712
 Sine. *See also* Law of Sines
 cofunction identity, 497
 defined, 506
 double-angle identity for, 587–588
 graphs of, 521–535
 amplitude and period, 524–527
 phase shift, 527–531
 simple harmonic motion, 633–635
 vertical shift, 531–533
 half-angle identity for, 592
 inverse, 547–548, 553
 period of, 521
 power-reducing identity for, 590
 product-to-sum identities and, 597–598
 properties of, 523
 right triangle definition, 493
 sum and difference identities for, 578–580, 584
 sum-to-product identities for, 599–600
 triple-angle identity for, 590
 unit circle definition, 515
 verifying trigonometric identities by converting to, 567–568, 570
 Sine curve, 522
 Sine wave, 522

- Single line, in degenerate conic section, 847
- Single logarithmic form, 446
- Sinusoid, 522
- Sinusoidal graphs/curves, 521–535
 - amplitude and period, 524–527
 - phase shift, 527–531
 - simple harmonic motion, 633–635
 - vertical shift, 531–533
- Slanted asymptotes, 369–370
- Slope-intercept form, of equation of a line, 196, 203
- Slope length, 503
- Slope of a line, 192–194
 - facts about, 194
- Smooth curve, 312
- Social Security numbers, 934, 936
- Solution(s)
 - of an inequality in two variables, 745
 - of equation, 84, 179
 - extraneous, 134, 135–136
 - of inequality, 147
 - of linear programming problem, 757
 - number of, of linear system, 728–731
 - optimal, 757
 - set of feasible, 757
 - of system of inequalities, 749
 - of system of linear equations
 - in three variables, 726
 - in two variables, 712–713
- Solution set, 84
 - of the inequality, 745
 - of a system of equations, 713
 - of system of inequalities, 749
- Solve, defined, 84
- Sonic cone, 544
- Sound waves, 574
- Space junk, 903
- Special products, 33–34
 - formulas, 36
- Speed
 - angular, 487–488
 - average, 487
 - hypersonic, 538
 - of light, 132, 141–142, 390
 - linear, 487–488
 - subsonic, 538
 - supersonic, 538
- Sphere, formulas for, 74
- Spiral of Archimedes, 687
- Spirals, 692
- Square matrix of order n , 785
- Square root function, 242, 244
- Square root method, for quadratic equation, 120
- Square Root Property, 120
- Square roots, 59–61
 - of negative numbers, 109
 - multiplication involving, 112
 - principal, 59
 - product property of, 61
 - quotient property of, 61
 - rationalizing denominators, 62–63
 - simplifying, 61–62
 - solving equations containing, 138
- Squares
 - difference of, 42–43
 - products giving sum or difference of, 36
- Squares (geometric), formulas for, 74
- Squaring
 - a binomial sum or difference, 34–35
 - solving trigonometric equation by, 610–611, 616–617
- Squaring function, 243
 - graphing, 232
- SSA triangles, 629
 - ambiguous case and, 637–639
 - solve possible, 639–641
- SSS (side-side-side) Theorem, 72
- SSS triangles spell out?, 629, 630
 - area of, 654–655
 - Heron's formula for, 654
 - Law of Cosines and, 644
 - solving, 647–648
- Standard equation of parabola with vertex $(0,0)$ and focus $(a,0)$, 850
- Standard form
 - of complex number, 109
 - of conditional linear equation in one variable, 85
 - of equation of a circle, 185, 187
 - of equation of a hyperbola, 871, 872
 - of equation of an ellipse, 860–861, 865
 - of polynomial, 30
 - of quadratic equation, 118
 - of quadratic function, 301–303
- Standard position, of angle, 480
- Standard unit vectors, 663
- Statement P_{k+1} , *determining from statement P_k* , 922–924
- Statins, 221
- Statute miles, 490
- Steinmetz, Charles, 108, 695
- Step functions, 247
 - graphing, 247
- Stock, 738
- Stock market investment, 738, 741–742
- Straight angle, 480
- Stretching a function, 257–259, 260
 - exponential functions, 411
- Strings, 892
- Subset, 6
- Subsonic speed, 538
- Substitution method
 - for squaring a binomial sum, 35

- for system of linear equations in two variables, 715–716
- for systems of nonlinear equations, 739
- Subtraction
 - of complex numbers, 110–111
 - of fractions, 13–14
 - matrix, 802–803
 - of polynomials, 31–32
 - of radical expressions, 64–65
 - of rational expressions, 51–54
 - with different denominators, 53
 - with same denominator, 51
 - when denominators have no common factor, 52
 - of real numbers, 12–14
 - vector, 659
- Sum
 - of arithmetic sequence, 905–907
 - of finite geometric sequence, 913–914
 - of functions, 268
 - of infinite geometric series, 916–917
 - of matrices, 801
 - in summation notation, 898
- Sum and difference of terms, product of, 35–36
- Sum identities
 - for cosine, 574–576, 584
 - for sine, 578–580, 584
 - for tangent, 583, 584
- Summation (sigma) notation, 898–899
 - properties of, 899
 - writing partial sum in, 899–900
- Sum-to-product identities, 599–600
 - solving trigonometric equation using, 617–618
- Sun, distance of planets from, 637
- Supersonic speed, 538
- Supplementary angles (supplements), 71, 485
- Supply-side economics, 359
- Sylvester, James Joseph, 801
- Symbols
 - electric current, 109
 - equal, 3
 - grouping, 9
 - inequality, 5, 7, 146
 - radical sign, 59, 63
- Symmetric image, 182
- Symmetry, 182–185
 - even-odd functions and, 232–233
 - in polar coordinates, 687
 - with respect to the origin, 182, 184
 - with respect to the x -axis, 182
 - with respect to the y -axis, 182, 184
 - sketching graphs using, 184–185
 - tests for, 182
- Symmetry Property of Graphs of f and f^{-1} , 282
- Synthetic division, 329, 331–333
- System of equations, 379–380
- Systems of inequalities, 745–755
 - graph of linear inequality in two variables, 745–748
 - nonlinear inequality, 751
 - nonlinear systems, 752–753
 - systems of linear inequalities in two variables, 749–751
- Systems of linear equations
 - compared to augmented matrix, 788
 - inconsistent, 792
 - solving
 - using Cramer's Rule, 832
 - using Gaussian elimination, 790–792
 - using Gauss-Jordan elimination, 792–794
 - using matrices, 785–789
 - using matrix inverses, 818–820
 - in three variables, 725–737
 - application to CAT scans, 725, 732–733
 - Gaussian elimination, 726–728
 - geometric interpretation, 731–732
 - nonsquare systems, 731
 - number of solutions, 728–731
 - solving using Cramer's Rule, 833–834
 - in two variables, 712–724
 - applications, 719
 - elimination system for, 717–718
 - graphical method for, 713–714
 - solving using Cramer's Rule, 832
 - substitution method for, 715–716
- Systems of linear inequalities in two variables, 749–751
- Systems of nonlinear equations, 712, 738
 - applications, 741–742
 - solving
 - by elimination, 740–741
 - by substitution, 739
- T
- Tables, for functions, 212
- Tali*, 891
- Tangent
 - cofunction identity for, 497
 - defined, 506
 - double-angle identity for, 588
 - graphs of, 538–542
 - half-angle identity for, 592
 - inverse, 550, 553
 - periodicity of, 521
 - power-reducing identity for, 590
 - properties of, 540
 - right triangle definition, 493
 - sum and difference identities for, 583, 584
 - unit circle definition, 515
- Tangent hyperbolic function, 427
- Tangent line, 743
 - to ellipse, 869

- Tap-water flow, friction and, 59, 61–62
- Tartaglia, Nicolo, 351
- Technology matrix, 820
- Telescopes
 - Hubble, 848, 855
 - reflecting, 880
- Temperature
 - conversion between Celsius and Fahrenheit, 90–91, 153
 - finding, from cricket chirps, 15
- Terminal point, of vector, 658
- Terminating decimal, 3
- Terms
 - constant, 30, 311
 - defined, 14
 - leading, 30, 311
 - like, 30
 - product of sum and difference of, 35–36
 - of sequence, 892–893
- Test point, 747
- Test-point method, 376–381
- Theoretical probability, 951
- Theory of probability, 927
- 30°-60°-90° triangle, 496
- 3 × 3 determinant, 830–831
- Time, interest and, 417
- Time dilation, 132, 141–142
- Touch-tone phones, 597, 601
- Tower of Hanoi puzzle, 926
- Transcendental functions, 424
- Transcendental numbers, 424
- Transformations
 - combining, 256
 - in sequence, 260–261, 262–263
 - on exponential functions, 411–413
 - of functions, 251–266
 - horizontal shifts, 251–255, 262
 - multiple transformations in sequence, 260–261, 262–264
 - reflections, 255–256, 262
 - stretching or compressing, 257–259, 262
 - summary, 262–263
 - vertical shifts, 251–255, 262
 - logarithmic functions, 434–435
 - nonrigid, 257
 - rigid, 257
- Transitive Property, 5
- Translations
 - of ellipses, 863–865
 - of hyperbolas, 877–880
 - of parabolas, 853–854
- Transverse axis of hyperbola, 871, 872, 877
- Trapezoid, formulas for, 74
- Tree diagram, 935
- Triangle inequality, 648, 860
- Triangle(s), 71–72
 - AAS, 629, 631–634
 - area of, 653–654
 - altitude of, 630
 - angles of, 71
 - area of, 651
 - AAS, 653–654
 - ASA, 653–654
 - SAS, 651–653
 - SSS, 654–655
 - ASA, 629, 631–634
 - area of, 653–654
 - congruent, 72, 629
 - equilateral, 71, 495
 - formulas for, 74
 - 45°-45°-90°, 495
 - included angle, 72
 - included side, 72
 - isosceles, 71
 - labeling, 629
 - oblique, 629–630
 - Pascal's, 927, 928–932
 - perfect, 656
 - right, 72, 173
 - SAS, 629, 630
 - area of, 651–653
 - Law of Cosines and, 644–647
 - scalene, 71
 - Sierpinski's, 925–926
 - similar, 72, 493, 629
 - SSA, 629
 - ambiguous case, 637–639
 - solve possible, 639–641
 - SSS, 629, 630
 - area of, 654–655
 - Law of Cosines and, 644, 647–648
 - 30°-60°-90°, 496
- Triangle Sum Theorem, 71
- Triangulation, 629
- Trichotomy Property, 5
- Trigonometric equations, 604–622
 - approximate solutions, 606–607
 - containing inverse functions, 618–620
 - extraneous solutions, 610–611
 - of form $a \sin(x - c) = k$, $a \cos(x - c) = k$, $a \tan(x - c) = k$, 605–608
 - involving multiple angles, 614–617
 - linear, 607
 - with more than one trigonometric function, 609–610
 - quadratic, 609
 - solving
 - using identities, 609–610
 - using squaring, 610–611, 616–617
 - using sum-to-product identities, 617–618
 - zero-product property and, 608–609
- Trigonometric form, of complex number, 696–698

- Trigonometric function applications, 628–710
 - area of a triangle, 651–657
 - dot product, 670–679
 - Law of Cosines, 644–650
 - Law of Sines, 629–636
 - ambiguous cases, 637–643
 - polar coordinates, 680–694
 - polar forms of complex numbers, 695–705
 - vectors, 658–669
 - Trigonometric functions, 478–562
 - of acute angle, 492–495
 - amplitude and period, 524–527
 - of angles, 506–507
 - angles and their measure, 479–491
 - composition of inverse trigonometric functions
 - and, 553–555
 - cosecant function graphs, 542–545
 - cosine function graphs, 521–522
 - cosine function properties, 523
 - cotangent function graphs, 538–542
 - of coterminal angles, 509
 - inverse, 547–556, 618–620
 - periodic functions, 520–521
 - phase shift, 527–531
 - of quadrantal angles, 508–509
 - of real numbers, 514–515
 - reference angle, 510–514
 - right triangle trigonometry, 492–505
 - secant function graphs, 542–545
 - signs of, 509–510
 - simple harmonic motion, 533–535
 - sine function graphs, 521–522
 - sine function properties, 523
 - tangent function graphs, 538–542
 - unit circle and, 514–517, 522–523, 564
 - vertical shifts, 531–533
 - visualizing, 564
 - Trigonometric identities, 563–627. *See also*
 - Trigonometric equations
 - alternating current, 591–592
 - analyzing touch-tone phones, 601
 - cofunction, 497, 577–578, 584
 - difference identities
 - for cosine, 574–576, 584
 - for sine, 578–580, 584
 - for tangent, 583, 584
 - double-angle identities, 587–590
 - even-odd, 565–566
 - fundamental, 497–499, 565
 - half-angle, 592–594
 - power-reducing identities, 590–591
 - product-to-sum, 597–598
 - Pythagorean, 498, 565
 - quotient, 498, 565
 - reciprocal, 498, 565
 - reduction formula, 581–582, 584
 - solving trigonometric equations using, 609–610
 - sum identities
 - for cosine, 574–576, 584
 - for sine, 578–580, 584
 - for tangent, 583, 584
 - sum-to-product, 599–600
 - sum-to-product identities, 617–618
 - verifying, 564–573, 600–601
 - methods, 566–570
 - Trigonometric ratios, 492–495
 - Trinomials, 30
 - factoring
 - of form $Ax^2 + Bx + C$, 44–45
 - of form $x^2 + bx + c$, 40–41
 - perfect-square, 42, 121
 - Triple-angle identity, 590
 - Trough, 220
 - Turning points, 231, 320
 - 2×2 determinant, 827–828
 - 2×2 matrix, finding inverse of, 818
- U**
- Ulam conjecture, 902
 - Unbounded interval, 7
 - Underwater devices, water pressure on, 278, 286
 - Uniform motion problems, 96, 99–101, 101
 - Union of sets, 6
 - Unit circle, 186
 - trigonometric functions and, 514–517, 564
 - using definition of, 522–523
 - Unit vectors, 662–663
 - Upper bound, 344
 - Upper limit, of summation, 898
- V**
- Value
 - of the algebraic expression, 14
 - of annuity, 914, 915
 - of arithmetic expression, 9
 - of f at the number, 212
 - Vandermonde, Alexandre-Théophile, 827
 - Variable
 - defined, 2
 - dependent, 211
 - independent, 211
 - polynomials in one, 30
 - set-builder notation and, 6
 - solving for specified, 91
 - Variable cost, 222
 - Variation
 - constant of, 387
 - direct, 386–388
 - inverse, 389–390
 - joint and combined, 390–392
 - of sign, 343–344
 - solving variation problems, 392

Varies directly, 387
Varies inversely, 389
Vector quantity, 658
Vectors, 658–669
 addition of, 659–660
 algebraic, 660–662
 applications, 665–667
 decomposition of, 676–677
 equivalent, 659
 geometric, 658–660
 in \mathbf{i} , \mathbf{j} form, 663
 opposite, 659
 orthogonal, 673–674
 parallel, 671–672
 position, 660–661
 projection of, 674–676
 resultant, 659
 standard unit, 663
 subtraction of, 659
 in terms of magnitude and direction, 658, 664–665
 unit, 662–663
 velocity, 666
 zero, 659
Velocity, 386
 vectors, 666
 wind, 658
Verdi, John, 146, 651
Verhulst, Pierre François, 456, 464
Verhulst model, 456
Vertex (vertices)
 of angle, 479
 of ellipse, 859, 860, 861, 863
 of hyperbola, 871, 872, 873, 877
 of parabola, 301, 849, 850, 853
 of right circular cone, 846
 of solution set, 750
Vertical asymptotes, of rational function, 361–364
Vertical ellipse, 863
Vertical lines
 equation of, 196–197, 203
 slope of, 194
Vertical-line test, 216
Vertical rise, 503
Vertical shifts, 251–255, 260, 262, 531–533
 on exponential functions, 411
Vertical stretch and compression, 257–258, 262
 on exponential functions, 411
Viewing rectangle, 173
Vocabulary, polynomial, 29–31
Voltage (V), 587
Volts, 587
Volume
 formulas, 74
 of wine barrel, 311, 322

W
Water pressure, on underwater devices, 278, 286
Watt, James, 745
Wattage rating, 587, 591–592
Watt (W), 587
Waves, sine, 522
Whispering galleries, 866
Whole numbers, 3, 4
Wind velocity, 658
Wine barrel, volume of, 311, 322
Wizard of Oz, 71
Work, measuring, 670, 677
Worker bee, 892
Work-rate problems, 101–103

X
x-axis
 defined, 170
 reflection in the, 255–256, 262
 symmetric with respect to, 182
x-coordinate, 170
x-intercept, 181
xy-plane, 170

Y
y-axis
 defined, 170
 reflection in the, 255–256, 262
 symmetric with respect to, 182, 184
y-coordinate, 170
y-intercept, 181, 196
Yours, Mine, and Ours, 300, 305

Z
Zero. *See also* Complex zeros of polynomial functions; Real zeros of polynomial functions
 of multiplicity, 319
Zero factorial, 896
Zero integer exponents, 20
Zero matrix, 803
 0_{mn} , 803
Zero polynomial, 30
Zero-Product Property, 11
 trigonometric equations and, 608–609
Zeroth row, of Pascal's triangle, 929
Zero vector, 659